Maxmin expected utility

Theory of Individual and Strategic Decisions

M.Sc. in Human Decision Science, Maastricht University

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Roadmap

1. Violations of Subjective Expected Utility
2. Relaxing the Independence of Irrelevant Alternatives
3. Maxmin Expected Utility

Subjective expected utility theorem

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  (A_3'') Continuity
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    - (A_6'') Non-triviality
  - There exist
    - a utility function $u : Z \to \mathbb{R}$
    - a unique belief $\mu \in \Delta(\Omega)$

    such that $\succeq$ are represented by

$$U(f) := \sum_{\omega \in \Omega} \mu(\omega) \cdot \left(\sum_{z \in Z} f(\omega)(z) \cdot u(z) \right)$$

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- Write them as acts:

act	2 black/0 yellow (ω_1)	1 black/1 yellow (ω_2)	0 black/2 yellow (ω_3)
f_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$
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- This is not consistent with SEU.

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- $f_1 \succ f_2 \Leftrightarrow 2\mu(\omega_1) + \mu(\omega_2) < 1$
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- Thus, $f_1 \succ f_2$ and $g_2 \succ g_1$ cannot occur simultaneously.

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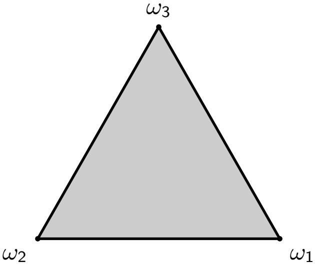

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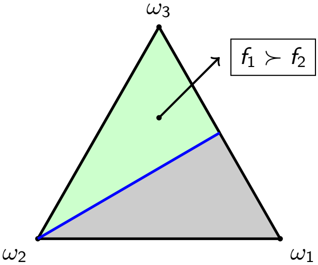

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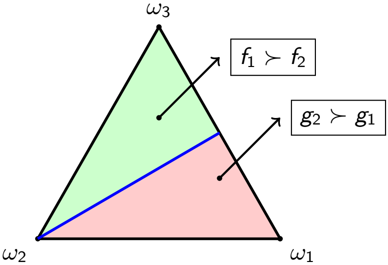
g_2	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$

- $f_1 \succ f_2 \Leftrightarrow 2\mu(\omega_1) + \mu(\omega_2) < 1$
- $g_2 \succ g_1 \Leftrightarrow 2\mu(\omega_1) + \mu(\omega_2) > 1$



Violation of SEU: The Ellsberg paradox

act	2 black/0 yellow (ω_1)	1 black/1 yellow (ω_2)	0 black/2 yellow (ω_3)
f_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$
f_2	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1 \circledast 0)$
g_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(1 \circledast 10)$
g_2	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$

- Suppose that IIA holds:

Violation of SEU: The Ellsberg paradox

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f_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$
f_2	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1 \circledast 0)$
g_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(1 \circledast 10)$
g_2	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$

- Suppose that IIA holds:
  - $f_1 \succ f_2 \Leftrightarrow (0.5 \circledast f_1, 0.5 \circledast g_1) \succ (0.5 \circledast f_2, 0.5 \circledast g_1)$

Violation of SEU: The Ellsberg paradox

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f_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$
f_2	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1 \circledast 0)$
g_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(1 \circledast 10)$
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act	2 black/0 yellow (ω_1)	1 black/1 yellow (ω_2)	0 black/2 yellow (ω_3)
h_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$
h_2	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$

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act	2 black/0 yellow (ω_1)	1 black/1 yellow (ω_2)	0 black/2 yellow (ω_3)
h_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$
h_2	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$

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f_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$
f_2	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1 \circledast 0)$
g_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(1 \circledast 10)$
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h_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$
h_2	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$

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- $(0.5 \circledast f_2, 0.5 \circledast g_1) = h_2 = (0.5 \circledast g_2, 0.5 \circledast f_1)$

Violation of SEU: The Ellsberg paradox

act	2 black/0 yellow (ω_1)	1 black/1 yellow (ω_2)	0 black/2 yellow (ω_3)
f_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$
f_2	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1 \circledast 0)$
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act	2 black/0 yellow (ω_1)	1 black/1 yellow (ω_2)	0 black/2 yellow (ω_3)
h_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$
h_2	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$

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- Hence, we need to relax IIA to explain this data. Otherwise:

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f_2	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1 \circledast 0)$
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g_2	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$

- Suppose that IIA holds:
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act	2 black/0 yellow (ω_1)	1 black/1 yellow (ω_2)	0 black/2 yellow (ω_3)
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act	2 black/0 yellow (ω_1)	1 black/1 yellow (ω_2)	0 black/2 yellow (ω_3)
f_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/3 \circledast 10, 2/3 \circledast 0)$
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h_1	$(1/3 \circledast 10, 2/3 \circledast 0)$	$(1/2 \circledast 10, 1/2 \circledast 0)$	$(2/3 \circledast 10, 1/3 \circledast 0)$
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Roadmap

Gilboa-Schmeidler preferences

Definition

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(A_5^*) **Monotonicity**: For any two acts $f, g \in \mathcal{F}$ with $f(\omega) \succeq g(\omega)$ for all $\omega \in \Omega$ it is the case that $f \succeq g$.

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(A_7^*) **Ambiguity aversion**: For any two acts $f, g \in \mathcal{F}$ with $f \sim g$ and any $\alpha \in [0, 1]$ it is the case that $\big(\alpha \circledast f, (1-\alpha) \circledast g\big) \succeq f$.

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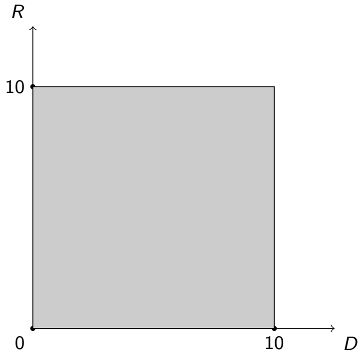

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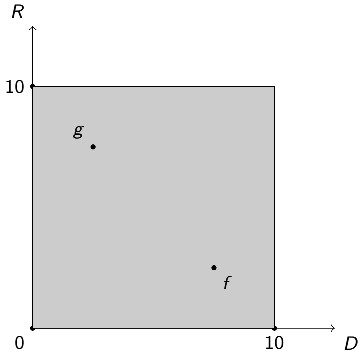
$$f \succeq g \Leftrightarrow (0.5 \circledast f, 0.5 \circledast h) \succeq (0.5 \circledast g, 0.5 \circledast h)$$



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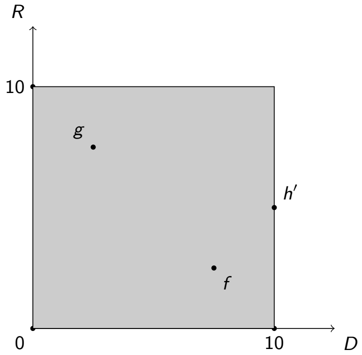

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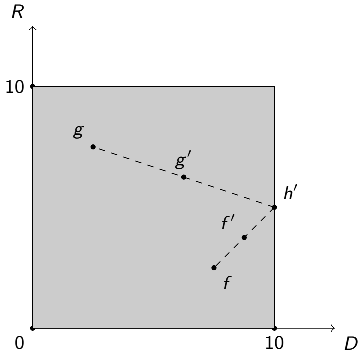

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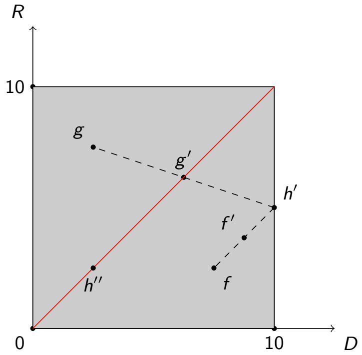
- **C-independence** (for all $h'' \in \mathcal{F}_c$)



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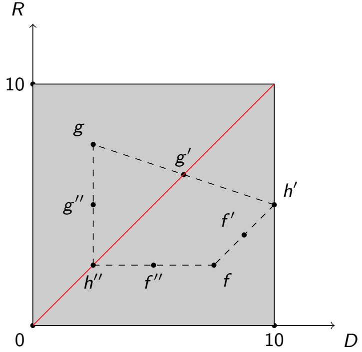

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  - If f is chosen the probability of each outcome is not known.

Roadmap

Maxmin expected utility theorem

Theorem (Gilboa & Schmeidler, 1989)

Consider a finite set of outcomes Z and a finite state space Ω. Then, the preferences $\succeq$ over $\mathcal{F}$ satisfy $A_1^* - A_7^*$ if and only if there is a function $u : Z \to \mathbb{R}$ and a closed and convex set of probability measures $C \subseteq \Delta(\Omega)$ such that for every $f, g \in \mathcal{F}$,

$$f \succeq g \Leftrightarrow \min_{\mu \in C} \sum_{\omega \in \Omega} \mu(\omega) \left(\sum_{z \in Z} f(\omega)(z) u(z) \right) \geq \min_{\mu \in C} \sum_{\omega \in \Omega} \mu(\omega) \left(\sum_{z \in Z} g(\omega)(z) u(z) \right)$$

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The **maxmin expected utility** function that represent $\succeq$ is

$$\boxed{U_M(f) := \min_{\mu \in C} \sum_{\omega \in \Omega} \mu(\omega) \cdot \left(\sum_{z \in Z} f(\omega)(z) \cdot u(z) \right)}$$

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- Finally, the MEU is the worst-case scenario among all possible beliefs in C.

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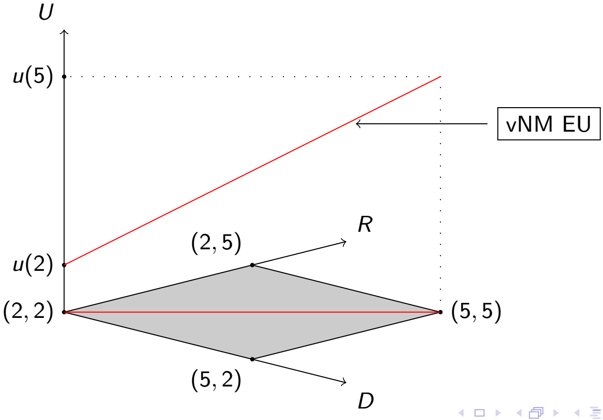

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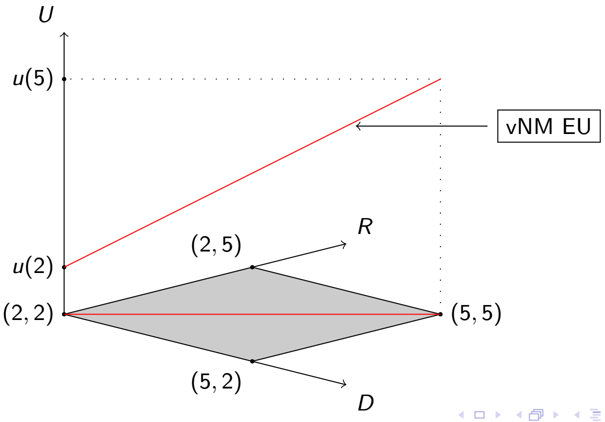

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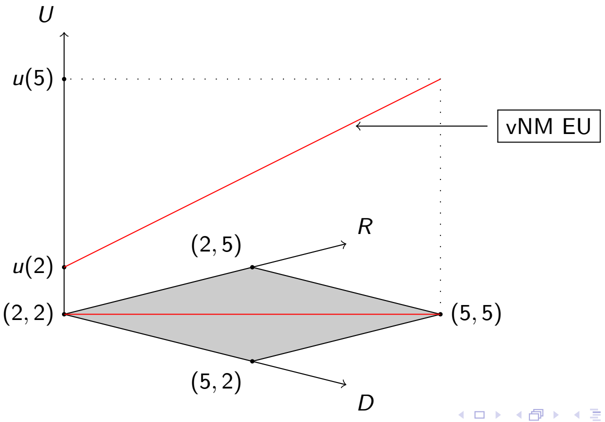

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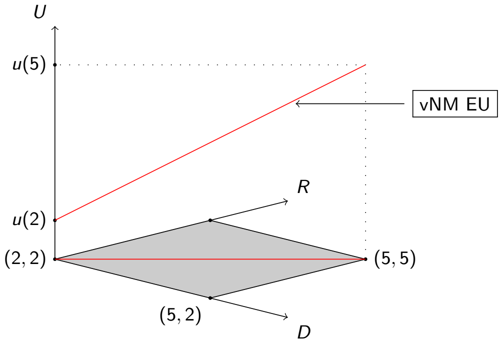

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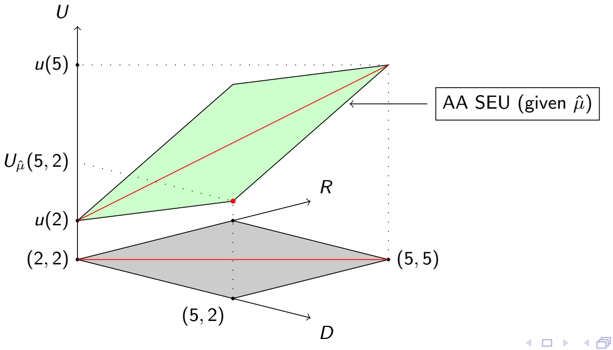

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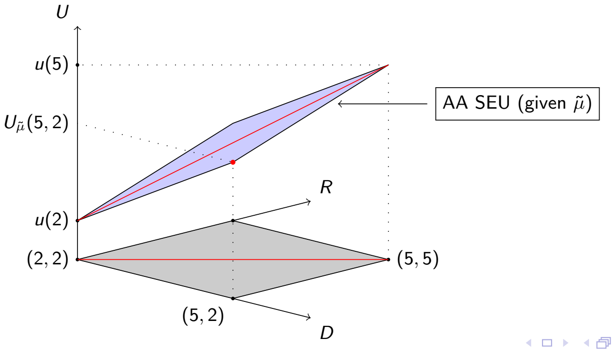

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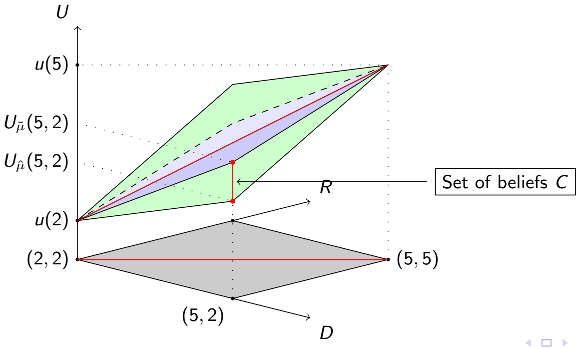
- The set of planes corresponds to the set of beliefs:

$$C = \{\mu \in \Delta(\Omega) : \hat{\mu}(D) \leq \mu(D) \leq \tilde{\mu}(D)\}$$



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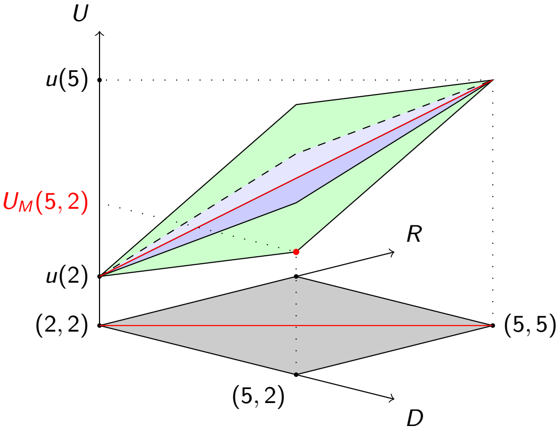

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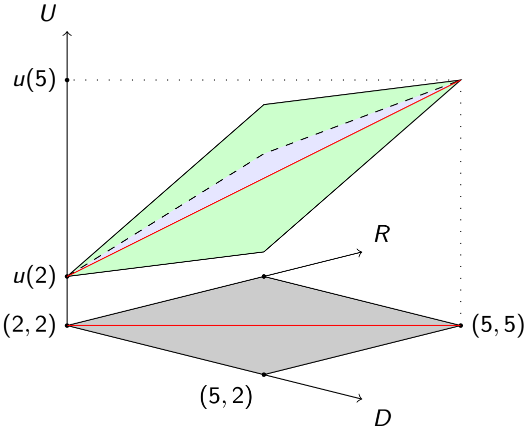

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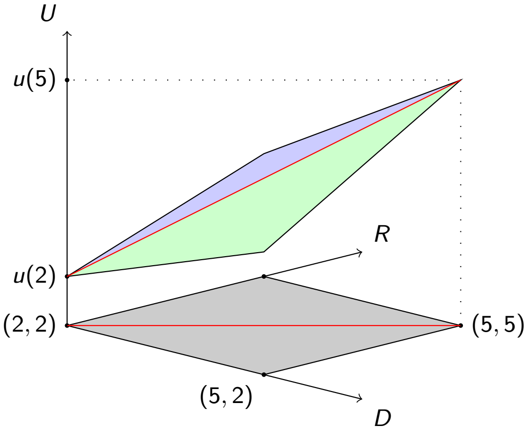

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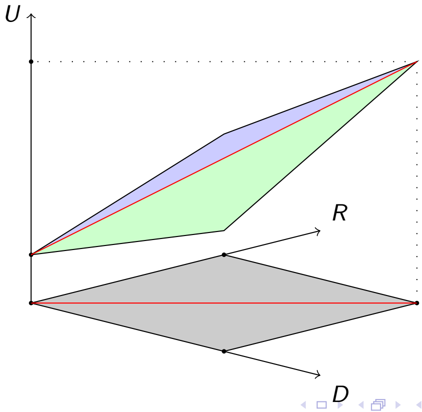

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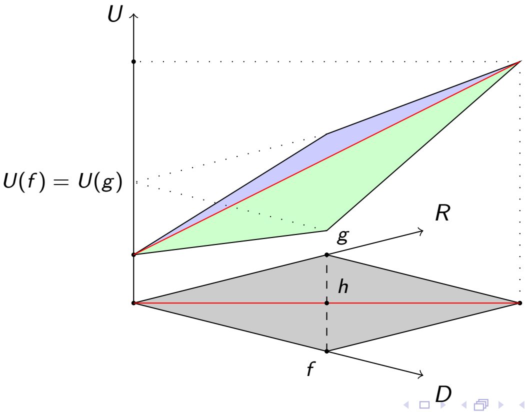

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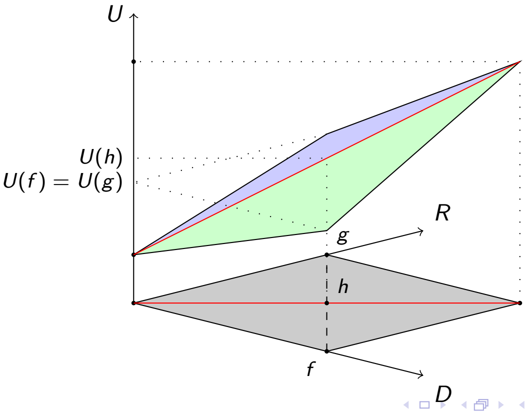

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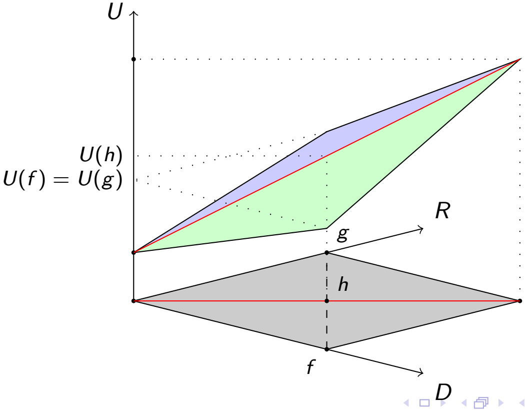
- This is because of ambiguity aversion: $f \sim g \Rightarrow h \succeq f$.



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- Hence, $f_1 \succ f_2$ and $g_2 \succ g_1$.

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Contribution of GS

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  - There are alternative models of ambiguity in the literature.

Thanks for watching!!!